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SUPERSONIC FLOW AROUND BLUNT BODIES OF REVOLUTION WITH A GENERATRIX DISCONTINUITY

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SUPERSONIC FLOW AROUND BLUNT BODIES OF REVOLUTION WITH A GENERATRIX DISCONTINUITY

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The integral method of Vaglio-Laurin is used to solve the supersonic flow problem around spherical segments. It is shown that second and third order terms are unimportant.

A calculation of supersonic flow around bodies with a generatrix discontinuity is of great practical importance, since the presence of a discontinuity decreases the heat exchange close to the breaking point for a given radius of curvature for the forward part. However, additional difficulties related to the occurrence of a singularity at the discontinuity point arise when calculating flow around bodies with a sonic discontinuity. The zone in which the flow turns around the corner point is located in the region of mixed flow. The flow turning is accompanied by a change in the flow velocity, both in terms of magnitude and direction. A "suspended" condensation jump may occur in the supersonic region around the lateral surface of the body. This article solves the problem of calculating the supersonic flow around blunt bodies with a discontinuity which determines the position of the sonic point on the body. Version II of the integral relationship method (Ref. 1) is investigated. This version employs the asymptotic solution of Vaglio-Laurin (Ref. 2), which has been refined and which leads to a form which is suitable for computer calculations. The results derived from calculating the flow around spherical segments ($\kappa = 1.4$, $M_\infty = 10$) are presented. /930*

The reverse problem of supersonic flow around a body with a discontinuity was investigated in (Ref. 3). The study (Ref. 4) also presents certain results derived from calculating flow around bodies with a discontinuity according to the method advanced by G. F. Telenin.

2. Let a supersonic current of ideal gas encounter an axisymmetric body at zero angle of attack. We shall investigate the case when the flow reaches the local speed of sound at the discontinuity point, and the form of the body behind the discontinuity has no influence on the subsonic flow close to the leading section. We must find the solution in the region delineated by the body surface, the shock wave, the axis of symmetry, and the limiting characteristics. Closure of the integration region from above by the limiting characteristics enables us to make a correct determination of the influence of the region.

Let us introduce the coordinate system s, $\xi = 1 - n/\varepsilon(s)$, where ε is the distance along the normal from the body surface to the shock wave. The initial system of equations of gas dynamics consists of the equation of continuity and the equations of motion written in a divergent form:

* Numbers in the margin indicate pagination in the original foreign text.

$$\frac{\partial t\varepsilon}{\partial s} + \frac{\partial Q}{\partial \xi} = 0, \qquad \frac{\partial z\varepsilon}{\partial s} + \frac{\partial F}{\partial \xi} + Y\varepsilon = 0,$$
$$\frac{\partial f\varepsilon}{\partial s} + \frac{\partial L}{\partial \xi} + X\varepsilon = 0, \tag{1}$$

where $t = \rho vr,\ z = \rho uvr,\ f = (p + \rho v^2)r,\ Q = rAU - (1+\xi)\varepsilon_*'t,\ A = 1 + (1-\xi)\varepsilon / R,$
$F = rAH - (1-\xi)\varepsilon_*'z,\ r = r_0 + (1+\xi)\varepsilon\cos\theta,\ L = Az - (1+\xi)\varepsilon_*'f,\ l = \rho u,\ Y = -f/R + Ap\cos\theta,\ H = p + \rho u^2,\ X = -z/R + Ap\sin\theta,\ \varepsilon_*' = d\varepsilon / ds;$

R is the body radius of curvature at an arbitrary point; θ - the angle formed by the tangent to the body profile and the direction of the incoming flow ; r - distance from the body axis of symmetry; u, v - velocity components in the direction n, s, pertaining to the maximum velocity w_{max}; ρ - density pertaining to density of the incoming flow ρ_∞; p - pressure pertaining to $\rho_\infty w_{max}^2$; the linear dimensions pertain to the body radius of curvature at the critical point. /931

The solution must be satisfied by specific boundary conditions on the shock wave (in the case of ξ = 0), on the body (in the case of ξ = 1), and on the axis of symmetry s = 0. In addition, we must take into account the condition of compatibility along the characteristics. These boundary conditions and the additional condition of "splicing" together with the asymptotic solution in the vicinity of the corner completes the initial system of equations (1).

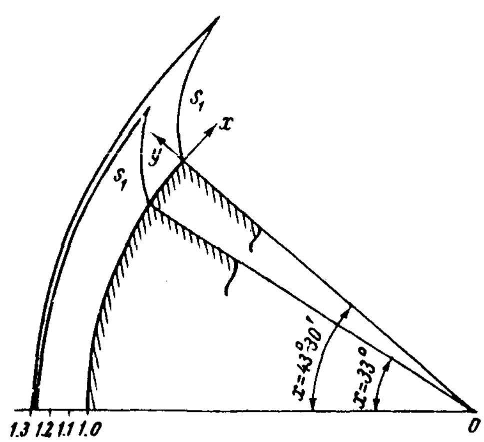

Figure 1

3. A solution which describes the plane and axisymmetric transsonic flow in the vicinity of the corner has the following form

$$U - 1 = (\varkappa + 1)^{-1/3} \sum_{i=0}^{\infty} \bar{n}^{(2+i)/4}\tilde{u}_i(\zeta),$$
$$V = \sum_{i=0}^{\infty} \bar{n}^{(3+i)/4}\tilde{v}_i(\zeta), \tag{2}$$
$$\zeta = (\varkappa + 1)^{-1/3}\bar{s}\bar{n}^{-5/4},\quad \bar{n} = n / r_{0*}$$
$$\bar{s} = (s - s_*) / r_{0*},$$

where U, V are the velocity components (pertaining to the critical flow velocity) along the tangent and the normal to the body surface at the corner point (Figure 1). The symbol * designates the values at the sonic point on the body. The functions $\tilde{u}_i$, $\tilde{v}_i$ are the solutions of the customary differential equations. The main term in (2) describes the two-dimensional potential flow* in the vicinity of the corner (the prime signs designate

* As follows from (Ref. 5), the function g corresponding to the potential of this flow may be expressed in parametric form as follows:

$$g = {}^{25}/_{42}\cdot 5^{-1/8}(5x^2 + 5x - 4)(1 - x)^{-7/8}(1 + {}^{3}/_{5}x)^{-9/8}C^{-3},$$ (continued on page 3)

derivatives with respect to ζ):

$$\tilde{u}_0 = g', \qquad \tilde{v}_0 = (7g - 5\zeta g')/4,$$
$$g'' = {}^1/_{16}B_1(21g - 25\zeta g'), \qquad 1/B_1 = g' - 25\zeta^2/16. \tag{3}$$

The system of differential equations for terms up to the third order are as follows:

$$\begin{gathered}
\tilde{u}_1' = B_1[\tilde{v}_1 + \nu \sin\theta_* - \tilde{u}_1(\tilde{u}_0' + 15\zeta/16)], \\
\tilde{v}_1' = B_1[{}^1/_4\, \tilde{u}_1(3\tilde{u}_0 + 5\zeta\tilde{u}_0') - {}^5/_4\,\zeta(\tilde{v}_1 + \nu\sin\theta_*)], \\
\tilde{u}_2' = B_1\{{}^5/_4\,\zeta[\nu K(\varkappa+1)^{1/3} - \tilde{u}_2] - B_2\}, \\
\tilde{v}_2' = B_1\{{}^5/_4\,\zeta B_2 - \tilde{u}_0[\nu K(\varkappa+1)^{1/3} - \tilde{u}_2]\}, \\
\tilde{u}_3' = -B_1(B_3 + {}^{25}/_{16}\zeta\tilde{u}_3), \qquad \tilde{v}_3' = {}^5/_4 B_1(\tilde{u}_0\tilde{u}_3 + \zeta B_3),
\end{gathered} \tag{4}$$

where

$$\begin{gathered}
B_2 = -{}^5/_4\tilde{v}_2 + \tilde{u}_0'\tilde{u}_2 + \tilde{u}_1'\tilde{u}_1 + (\varkappa+1)^{-1/3}\{{}^1/_4(3\varkappa-1)\tilde{u}_0\tilde{v}_0' + {}^1/_2\tilde{u}_0^2\tilde{u}_0' + \tilde{v}_0\tilde{v}_0' - \\
- {}^5/_4\,\zeta[(\varkappa-1)\tilde{u}_0\tilde{v}_0' + \tilde{u}_0'\tilde{v}_0]\}, \\
B_3 = -{}^3/_2\tilde{v}_3 + \tilde{u}_0'\tilde{u}_3 + \tilde{u}_1'\tilde{u}_2 + (\varkappa+1)^{-1/3}\{{}^1/_2(2\varkappa-1)\tilde{u}_0\tilde{v}_1 + {}^3/_4\varkappa\tilde{u}_1\tilde{v}_0 + {}^1/_2\tilde{u}_0^2\tilde{u}_1' + \\
+ \tilde{u}_0\tilde{u}_0'\tilde{u}_1 + \tilde{v}_0'\tilde{v}_1 + \tilde{v}_0\tilde{v}_1' + \nu(\varkappa-2)\tilde{u}_0\sin\theta_* - {}^5/_4\zeta\{\tilde{u}_1'\tilde{v}_0 + \tilde{u}_0'\tilde{v}_1 + \\
+ (\varkappa-1)(\tilde{u}_0\tilde{v}_1' + \tilde{u}_1\tilde{v}_0')]\},
\end{gathered}$$

/932

$$K = \rho_* w_* a r_{0*}^2 \left[\frac{2}{(\varkappa+1)^2 - (\varkappa-1)^2 a} - \frac{a-1}{2a}\right] \frac{\varkappa+1}{\varkappa-1}\left(\frac{\partial^2 v}{\partial r^2}\right)_{\zeta=0}^{r=0}.$$
$$a = 1 + 2/(\varkappa-1)M_\infty^2.$$

The quantity ν = 0 or 1, respectively, for the plane and axisymmetric cases.

If the equation for the body surface in the subsonic region has the following form (Figure 1)

$$y = a_1x^2 + a_2x^3 + \dots .$$

then the terms of the first order equal

$$\tilde{u}_1 = 0, \qquad \tilde{v}_1 = -\nu\sin\theta_*.$$

In the plane case (ν = 0) $\tilde{u}_3 = 0$, $\tilde{v}_3 = 0$. Boundary conditions for terms of the second and third orders may be written as follows:

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* (continued from page 2)

$$\zeta = -2\cdot 5^{-3/8}x(1-x)^{-5/8}\left(1 + \frac{3}{5}x\right)^{-3/8}C; \qquad -\frac{5}{3} < x < 1,$$

where $g'(1) = 0$ in the case of $C = 1$, C - scale constant. If $C = 1$, then $g \sim 125/56\cdot 2^{-1/5}(-\zeta)^{7/5}$ for $\zeta \to \infty$, $g \sim \zeta^3/3 - 675/96\cdot 10^{-1/3}\zeta^{1/3}$ in the case of $\zeta \to +\infty$.

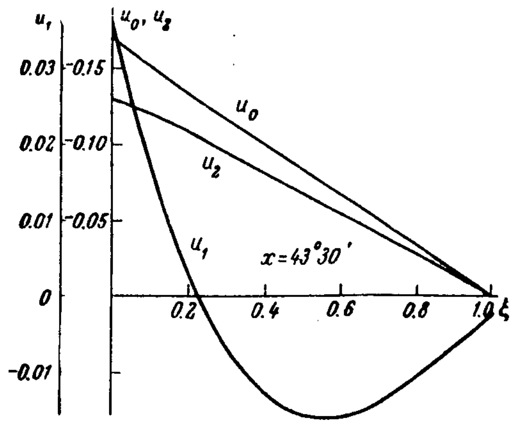

Figure 2

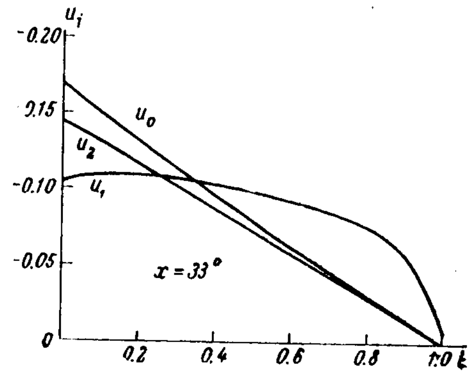

Figure 3

$$\zeta \to -\infty \begin{cases} \tilde{u}_2 = \sum\limits_{i=0}^{\infty} \gamma_i^{(1)}(-\zeta)^{4(1-i)/5}, & \tilde{v}_2 = \sum\limits_{i=0}^{\infty} \gamma_i^{(2)}(-\zeta)^{(5-4i)/5}, \\ \tilde{u}_3 = \sum\limits_{i=0}^{\infty} \gamma_i^{(3)}(-\zeta)^{-(3+8i)/5}, & \tilde{v}_3 = \sum\limits_{i=0}^{\infty} \gamma_i^{(4)}(-\zeta)^{(2-8i)/5}; \end{cases}$$

$$\zeta \to +\infty \begin{cases} \tilde{u}^2 = \gamma_0^{(5)}\zeta^4 + \gamma_1^{(5)}\zeta^{4/3} + \gamma_2^{(5)}\zeta^{-4/9} + \gamma_3^{(5)}\zeta^{-4/3} + \gamma_4^{(5)}\zeta^{-28/9} + \gamma_5^{(5)}\zeta^{-4} + \dots, \\ \tilde{v}_2 = \gamma_0^{(6)}\zeta^5 + \gamma_1^{(6)}\zeta^{7/3} + \gamma_2^{(6)}\zeta + \gamma_3^{(6)}\zeta^{-5/9} + \gamma_4^{(6)}\zeta^{-1/3} + \gamma_5^{(6)}\zeta^{19/9} + \dots, \\ \tilde{u}_3 = \sum\limits_{i=0}^{\infty} \gamma_i^{(7)}\zeta^{(3-8i)/3}, \quad \tilde{v}_3 = \sum\limits_{i=0}^{\infty} \gamma_i^{(8)}\zeta^{2(3-4i)/3}. \end{cases}$$

where $\gamma_i^{(j)} = \gamma_i^{(j)}(C, \kappa, K, \theta_*)$, C - scale constant. It must be pointed out that terms $\tilde{u}_i$, $\tilde{v}_i$, $i = 2k + 1$, $k = 0, 1, 2, \dots$, are omitted in the solution presented in (Ref. 2), and the right hand sides of the differential equations for the terms beginning with the second order are not written correctly. The study (Ref. 6) presents the asymptotic expressions for U, V which correspond to the flow around the plane forward section ($1/R = 0$ for $s < s_*$) and corresponding to terms up to the second order.

The solution of (2) makes it possible to determine all of the requisite gasodynamic parameters both for the plane and the axisymmetric cases in the vicinity of the sonic discontinuity, within an accuracy of the scale constant C, for a specific form of the shock waves.

4. Version II of the integral relationship method is employed to provide a numerical solution of the system (1) of the partial differential equations in the region $0 < \xi < \xi_0$. In this method, the gasodynamic funtions

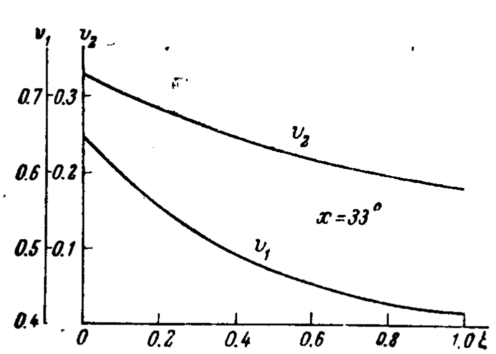

Figure 4

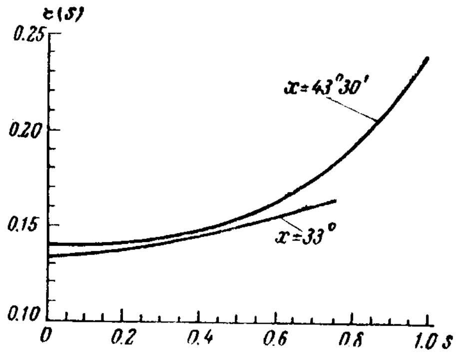

Figure 5

are approximated along s. The integration region is divided into bands by the lines $s_k = ks_1(\xi) / N$, $k = 1, 2, \ldots, N - 1$ ($s = s_1(\xi)$ - equation of the boundary characteristics). As is customary, the polynomial approximation with respect to s is employed, with allowance for the fact that the values of the desired functions for s = 0 are included in the approximating system.

In the vicinity of the corner, the field of flow may be calculated according to formulas (2) - (4).

The approximating system contains N + 1 unknown parameter $\varepsilon_{00}, \varepsilon_{10}, \ldots, \ldots, \varepsilon_{N0}$, where ε_{i0} - value of $\varepsilon(s)$ at the intermediate points $\xi = 0$. One additional unknown parameter -- the scale constant C -- is contained in the solution of (2). In all, we have N + 2 unknown parameters, which are determined from N boundary conditions on the body: $u_0, u_2, \ldots, u_N = 0$, in the case of $\xi = 1$ (u_i - value on the intermediate line) and two "splicing" conditions at the point $\xi = \xi_0$ in the vicinity of the corner. The problem is completely defined.

The computational procedure is as follows. In the case of $\xi = 0$, the initial values of the unknown functions are determined from the condition on the shock wave. After this, the approximating system is integrated up to the "splicing" point, where $\xi = \xi_0$. In the case of $\xi_0 < \xi < 1$, the approximating /933
system may be integrated concurrently with equations (2) - (4), which describe transonic flow in the vicinity of the corner. In order to determine the unknown parameters, the method of the steepest descent is employed. The accuracy with which the boundary value problem is solved is determined by the maximum permissible error, which may be given in the following form

$$\{(u_0^2 + u_2^2 + \ldots - u_N^2)_{\xi=1} + [(u_1 - u_{1v})^2 + (v_1 - v_{1v})^2]_{\xi=\xi_0}\}^{1/2} < \delta,$$

where u_{1v}, v_{1v} are the values of the velocity components at the point ξ_0 determined according to formula (2).

5. Let us present certain results derived from the calculations following the method indicated above ($M_\infty = 10$, $\kappa = 1.4$).

Figure 1 shows the form of the shock wave which is formed during flow around a spherical segment with the half opening of angle 43°30' and 33°. Figures 2-4 and the table present the velocity distribution on the axis of symmetry, the boundary characteristics $s = s_1(\xi)$, and on the intermediate line $s = s_1(\xi)/2$ (the quantities are designated by the indices 0, 1, and 2, respectively) in the case of $M_\infty = 10$, $\kappa = 1.4$. It may be seen that the behavior of u on the boundary characteristics greatly depends on the segment half opening angle. Figure 5 shows the law governing the change in the shock wave departure $\varepsilon(s)$ along the body.

As the calculational results show, an increase in the segment half opening angle leads to an increase in the scale constant C contained in the solution of (2). When the segment half opening angle corresponds to the position of the sonic point on the sphere, the scale constant becomes infinite -- i.e., the main term in (2) degenerates into the Prandtl-Meyer solution.

The calculations were performed in the second approximation (N = 2). A study of the algorithm of version II shows that the maximum error occurs at the boundary characteristics and does not exceed 2% in the case of /934
N = 2 (we should note that the velocity components at the "splicing point of

$æ = 33°$, $\varepsilon_{00} = 0.1332$, $\varepsilon_{10} = 0.1643$, $\varepsilon_{20} = 0.1436$, $C \sim 2.1$

i	ξ	s	u_i	v_i	i	ξ	s	u_i	v_i
	0		−0.171			0.6	0.617	−0.0943	0.452
	0.2		−0.132		1	0.8	0.593	−0.0791	0.428
0	0.4	0	−0.0965	0		1.0	0.576	−0.00705	0.415
	0.6		−0.0626			0	0.380	−0.148	0.329
	0.8		−0.0301			0.2	0.347	−0.119	0.281
	1.0		0			0.4	0.325	−0.0895	0.246
	0	0.759	−0.108	0.648	2	0.6	0.313	−0.0590	0.218
1	0.2	0.695	−0.111	0.548		0.8	0.297	−0.0279	0.195
	0.4	0.650	−0.106	0.490		1.0	0.288	0	0.177

the two solutions undergo a small discontinuity). The computational error on the axis and the mean line in the case of N = 2 comprises approximately 0.5%. The calculations show that the use of two terms in the solution of (2) is sufficient when determining the flow in the vicinity of the corner. Terms of the second and third order have no significant influence.

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